COGNITIVE NEUROPSYCHOLOGY, 2002, 19 (6), 505–522

REPRESENTING ARITHMETIC TABLE FACTS IN MEMORY: EVIDENCE FROM ACQUIRED IMPAIRMENTS

John Whalen
University of Delaware, Newark, USA

Michael McCloskey, Margarethe Lindemann, and Graham Bouton
Johns Hopkins University, Baltimore, USA

In this article we explore how arithmetic table facts (e.g., 6 × 9 = 54) are stored in memory and evaluate the possibility that arithmetic facts are exclusively stored in a phonological or sound-based form. We present two single-case studies of brain-damaged patients who suffer specific number processing impairments. Both patients often retrieve the correct answer to simple arithmetic problems from memory when unable to generate the phonological representation of either the arithmetic problem or the answer to that problem. We argue that this pattern of performance is incompatible with the hypothesis that arithmetic facts are stored and retrieved from memory exclusively in a phonological form. Accounts consistent with out findings are proposed.

THE PHONOLOGICAL STORAGE HYPOTHESIS

Acquiring skill in arithmetic involves, among other things, memorising the basic arithmetic table facts (e.g., 3 + 2 = 5; 6 × 7 = 42). However, the form in which the arithmetic facts are stored in memory is a matter of some controversy. One hypothesis holds that arithmetic facts are stored in a sound-based, or phonological, form (Cohen & Dehaene, 2000; Dehaene & Cohen, 1995; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999). One form of the *phonological storage hypothesis* suggests that arithmetic facts such as 6 × 8 = 48 are stored in memory phonologically (e.g., /sɪks taimz eit ɪz fordi eit/). This theory posits that arithmetic facts are accessed from memory using a phonological representation of the problem. For example, when we hear a problem (e.g., "six times eight"), the phonological representation (/sɪks taimz eit/) is used to address the memorised arithmetic fact (/sɪks taimz eit ɪz fordi eit/). The answer (/fordi eit/) can then be extracted from the retrieved arithmetic fact and converted to the appropriate form for output (e.g., 48, "forty-eight," FORTY EIGHT).

Initial support for this phonological storage hypothesis came from the finding that multilingual individuals solve arithmetic problems fastest in the language used when learning arithmetic (e.g., Kolers, 1968; Marsh & Maki, 1976; Shanon, 1984). This has been interpreted as evidence that arithmetic facts are stored phonologically in the language in which they were learned (Kolers, 1968; but see Noel & Seron, 1992). Frenck-Mestre and Vaid (1993) also report that response times for arithmetic verification interact with both the type

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Requests for reprints should be addressed to John Whalen, Department of Psychology, 200 Academy St #104, University of Delaware, Newark, DE, 19716, USA (Tel: (302) 831-0484; Email: whalen@udel.edu).

http://www.tandf.co.uk/journals/pp/02643294.html
DOI:10.1080/02643290244000086

of arithmetic foil problem (e.g., two + three = six) and the language in which it is presented (L1, L2), suggesting the possibility of language-specific arithmetic fact representations. However, speed differences in arithmetic fact retrieval across primary and secondary languages might reflect differences in numeral comprehension and production speeds across languages rather than differences in the form in which the facts were stored (Noel & Seron, 1992; but see Campbell, 1999). Differences in comprehension rates between languages might also affect the time course of interference effects from foils (e.g., two + three = six) reported during arithmetic verification tasks (Frenck-Mestre & Vaid, 1993).

Other evidence provided in support of language-based arithmetic fact representations comes from operand priming errors. Operand priming errors (e.g., problem: 8 × 4, erroneous response: 24), which occur more frequently when the problem is presented in a word-based format (e.g., /eit taimz fᴏʀ/, erroneous response: /twenti fᴏʀ/) relative to when it is presented in an Arabic form (Campbell, 1999; LeFevre, Qingwen, Smith-Chant, & Mullins, 2001). Campbell accounts for this finding in his encoding complex hypothesis by suggesting that one of the multiple representations of arithmetic facts is a phonological representation, and that the phonological representation is most active when the problem is presented in a word-based form (Campbell, 1994, 1997, 1998). However, data from Dutch and French participants performing arithmetic suggests that the source of these errors may lie in the manner in which the problem is comprehended and produced, rather than during the fact retrieval process (Brysbaert, Fias, & Noel, 1998; Noel, Fias, & Brysbaert, 1997; Noel, Robert, & Brysbaert, 1998), consistent with other studies of arithmetic fact retrieval that suggest there is an encoding difference, which may account for differences across stimulus formats rather than differences in the nature of the facts that are being retrieved (Blankenberger & Vorberg, 1997).

Finally, recent bilingual training studies are consistent with the notion that arithmetic facts may be stored in a language-based format. English-Russian bilinguals, trained on a unique set of unfamiliar arithmetic equations (e.g., 47 + 54 = ?) in each of their two languages, reveal answer latencies that differ across languages for exact arithmetic: Response times were fastest for problems tested in the language in which they had been trained. No such language-specific benefit was found for problems on which an approximate calculation was required (e.g., estimate cube root of 29). Based on these findings, Spelke and Tsivkin (2001) propose that linguistic representations of number facts may contribute to the representation of large exact numbers whereas approximation may be language independent.

Dehaene and Cohen (1995) propose a second form of the phonological storage hypothesis in which arithmetic facts are stored and retrieved using a level of representation in which there is a plan for verbal numeral production, but one in which phonological forms have not yet been inserted (such as Levelt's "lemma," 1989). For example, McCloskey, Sokol, and Goodman (1986) propose that verbal numeral production involves a syntactic plan that includes both a lexical stack choice (tens, teens, ones) and a position within the stack (1, 2, 3 ...) for each number word (e.g., Tens:{6}, Ones:{4} for "sixty-four"). Dehaene argues that "arithmetic facts are rote-memorised associations between filled word frame representations of the operands, rather than between their phonological representations" (1995, p. 109). In order to solve the problem of 6 × 4, it must first be converted into a verbal word frame, such as (Ones:{6} {Times} Ones:{4}, in order to retrieve the appropriate answer, which is also in a verbal word frame (Ones:{6} {Times} Ones:{4} Tens:{2}, Ones:{4}).

Dehaene and Cohen (1997; Cohen & Dehaene, 2000) further hypothesise that number facts learned by rote at school, such as the over-learned multiplication table, are retrieved as automatic verbal associations. In contrast, they propose that subtraction problems are not commonly learned by rote, and must be solved through mental manipulations of the quantities being represented, termed "semantic elaboration." It has also been proposed that elementary addition problems may also be solved using a verbal form, as in the case of multiplication; however, more complicated addition and also division problems may be solved through a number of quantity-based strategies (Cohen, Dehaene, Chochon, Lehericy, & Naccache, 2000).

The triple-code theory provides for a specific prediction, that there are limitations to the types of dissociations across operations consistent with the route used for calculation: Patients may have greater impairment of multiplication relative to addition due to impairments in fact retrieval, or greater impairment in subtraction relative to multiplication relative to subtraction due to impairments in the semantic elaboration process. However, this theory predicts that patients should never exhibit selective preservation or impairment of addition relative to multiplication and subtraction (Cohen & Dehaene, 2000). Consistent with this prediction are reports of selective impairments in multiplication relative to subtraction (Dagenbach & McCloskey, 1992; Dehaene & Cohen, 1997; Pesenti, Seron, & Vanderlinden, 1994), and selective impairment of subtraction relative to multiplication (Dehaene & Cohen, 1997; Delazer & Benke, 1997). Also consistent with the hypothesis that multiplication is mediated by verbal associations is the finding that several patients reveal an association of language disorders and impairment in multiplication (Dagenbach & McCloskey, 1992; Dehaene & Cohen, 1991, 1997; Delazer & Bartha, 2001; Delazer, Girelli, Semenza, & Denes, 1999; Girelli, Delazer, Semenza, & Denes, 1996; Hittmair-Delazer, Semenza, & Denes, 1994; Pesenti et al., 1994; Sokol, McCloskey, Cohen, & Aliminosa, 1991), whereas those with preserved multiplication tend to have intact linguistic skills (Dehaene & Cohen, 1997; Delazer & Bartha, 2001).

However, some cases reveal patterns of performance not predicted by the Cohen and Dehaene theory. For example, patients have been reported with intact calculation abilities despite severe aphasia (e.g., Rossor, Warrington, & Cipolotti, 1995) and severe impairment in arithmetic fact retrieval despite intact linguistic abilities (e.g., Warrington, 1982). Perhaps most challenging is the finding that multiple patients can have selective impairments multiplication, subtraction, or addition, one of which is not predicted by the triple-code model: Patient FS had selective impairment of addition relative to subtraction and multiplication (Van Harskamp & Cipolotti, 2001). Also lacking explanation is why a patient reported by Lampl, Eshel, Gilad, and Sarova Pinhas (1994) was severely impaired at performing simple addition (3/20) despite being able to complete complex subtraction. It is not clear why the semantic elaboration route was incapable of solving addition problems given that it was able to solve subtraction problems. Together, these appear to fit more closely with a theory of independent stores of arithmetic facts (Dagenbach & McCloskey, 1992; McCloskey, 1992; Van Harskamp & Cipolotti, 2001).

Hypotheses regarding the localisation of the multiplication and subtraction routines have generally matched the localisation of the injury sustained by brain-damaged patients and functional brain-imaging studies, but some exceptions have been reported. For example, whereas some patients reveal damage consistent with the prediction that multiplication and addition rely on the left parisylvian region as well as the basal ganglia, subtraction is thought to be implemented by a bilateral inferior parietal network (Dehaene & Cohen, 1997). Van Harskamp and Cipolotti's patients FS and VP suffered extensive damage to the inferior parietal area, yet had relative preservation of subtraction relative to impairment in addition and multiplication, and revealed intact basal ganglia and thalamic nuclei. Evidence from brain-imaging studies appears only somewhat consistent with the triple-code model. Chochon, Cohen, Van de Moortele, and Dehaene (1999) report bilateral inferior frontal gyrus activation when comparing multiplication to a letter naming task; however, no such difference was found in the activation of multiplication relative to digit naming task, while other regions such as the precentral gyrus and intraparietal sulcus were active in both comparisons (Cohen et al., 2000). Activation attributed to addition fact retrieval has been reported in left intraparietal and superior parietal areas as well as the precentral gyrus; however, no activation was found in language centres, which might have been expected (Pesenti, Thioux, Seron, & De Volder, 2000). Activation of the precentral gyri and intraparietal sulci have also been reported during multiplication verification (Rickard et al., 2000). Contrasts between multiplication and subtraction have revealed increased activation of left intraparietal sulcus during subtraction and increased activation of the right intraparietal region during multiplication. Fulbright et al. report that relative to a number matching task, a multiple multiplication task activates the left middle frontal gyrus and intraparietal sulci (Fulbright et al., 2000). However, the multiplication task used, which required three to four calculations resulting in a 3- or 4-digit number, clearly required fact retrieval and multi-digit computation procedures.

Predictions

According to both forms of the phonological storage hypothesis, arithmetic facts are stored exclusively in a phonological form (in either a phonological or a lemma representation) and addressed using a verbal representation of the arithmetic problem. Problems that are not initially represented internally in a phonological form (e.g., 4 × 6, IV × VI) *must* be converted into a phonological representation in order to address the appropriate arithmetic fact in memory.

A unique tenet of this theory is that the arithmetic fact retrieved from memory will correspond to the phonological encoding of the problem whether or not the problem is correctly transcoded into a phonological form. For example, if the problem 4 × 6 was successfully converted into the phonological form /fɔr taimz sɪks/ (as shown in Figure 1a), the arithmetic fact /fɔr taimz sɪks ɪz twɛnti fɔr/ should be retrieved from memory, assuming successful retrieval of the fact from memory. However, if an error is made in translating the problem into a phonological form, the answer retrieved from memory should reflect this error. As depicted in Figure 1b, if the problem 4 × 6 were mistakenly translated into the phonological representation /θri taimz sɪks/, then the arithmetic fact /θri taimz sɪks ɪz eitin/ should be retrieved from memory. Cohen and Dehaene (2000) describe a pattern of performance from a brain-damaged patient that appears consistent with this hypothesis. When presented with a multiplication problem in Arabic form (e.g., 5 × 9), the patient typically misnamed the problem ("four times six"), and then provided an answer consistent with her misnaming of the problem (24). In all other operations (e.g., 8 − 3), the patient almost always misnamed the problem ("six minus four"), but then provided the correct answer to the original problem (5). Cohen and Dehaene hypothesise that only multiplication answers were being retrieved from memory, whereas other answers were being computed using nonretrieval strategies (and therefore are not relevant to the predictions of the phonological storage hypothesis). This pattern of performance, in which responses to multiplication problems were consistent with spoken answers, has been reported in two other cases of impairment after brain injury (Girelli & Delazer, 1999; McNeil & Warrington, 1994).

Using the performance of two brain-damaged patients, we evaluate the predictions of the phonological storage hypothesis by comparing arithmetic facts retrieved from memory with the phonological representations of the problem. We consider both the Dehaene and Cohen position that multiplication facts may be stored in a language-based form tied to a lemma-like representation of arithmetic facts, and the more general hypothesis that arithmetic facts across several operations are represented in a phonological form.

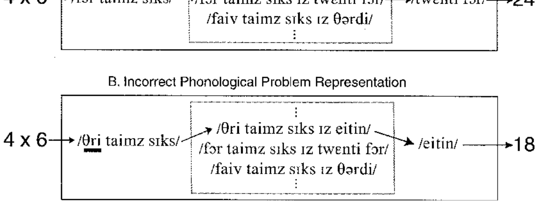


Figure 1. *Predictions of the phonological storage hypothesis: Arithmetic fact retrieval success depends on the phonological representation of the arithmetic problem.*

CASE 1: PATIENT KSR

Case history

KSR was a PhD candidate in chemical engineering before suffering a CVA in 1994 at age 44. He suffered a large infarct affecting the entire area of the middle cerebral artery distribution, including the left caudate, Broca's area, Wernicke's area, as well as the supramarginal gyrus (see Figure 2). Assessment with the Boston Diagnostic Aphasia Exam (Goodglass & Kaplan, 1983) revealed that KSR's ability to comprehend and produce spoken language were severely impaired, whereas written language comprehension and production were much less impaired (details provided in Appendix A).

Numeral comprehension and production. In evaluating the phonological storage hypothesis, we presented arithmetic problems in Arabic form (e.g., 8 × 6), and asked KSR to give responses in the form of written Arabic numerals (48) or spoken words ("forty-eight"). These tasks required (in addition to retrieval of stored arithmetic facts) comprehension of Arabic numerals, written production of Arabic numerals, and spoken production of numerals in word form. To assess KSR's ability to carry out these processes, we presented him with several transcoding tasks in which he translated numerals from one form (e.g., 6) to another (e.g., "six"). Stimuli in each transcoding task were ten numerals in the range 1–99 and ten in the range 100–99,999 from the Johns Hopkins Dyscalculia Battery (for detailed description of stimuli, see McCloskey, Aliminosa, & Macaruso, 1991). Additional tasks were also presented, as described next.

Arabic numeral comprehension. Results from several tasks revealed no impairments in Arabic numeral comprehension. KSR made no errors when transcoding Arabic numerals into written word numerals (20/20 correct: except a minor spelling error: 90 → ninty). KSR was also tested on magnitude comparison judgements, in which he was to choose the larger of two Arabic numerals in the same ranges as in the transcoding tasks. His magnitude comparison judgements were uniformly correct (20/20). Additional evidence of intact Arabic numeral comprehension comes from paper and pencil arithmetic, which required in part the comprehension of problems presented in Arabic numeral form. KSR solved 30 single-digit and 30 multi-digit addition, subtraction, and multiplication problems presented in Arabic form quickly and without error (60/60).

Arabic numeral production. Tests of Arabic numeral production also revealed no impairments. KSR made no errors when converting single- and multi-word written numerals into Arabic form (20/20). His ability to solve single- and multi-digit arithmetic problems that required written Arabic numeral answers (60/60) also indicated that Arabic numeral production was unimpaired.

Spoken numeral production. Transcoding tasks requiring spoken numeral production revealed significant impairments. KSR was only able to correctly name aloud 3 of 20 Arabic numerals, and 1 of 20 written numerals. Given that other evidence suggests that KSR can comprehend both Arabic and written word numerals, this pattern of performance suggests an impairment in spoken numeral production. KSR's pattern of errors reveals that he has the appropriate syntactic plan for producing the spoken numeral, but often selects the incorrect phonological representation for individual numerals. Including the arithmetic tasks described next, 77% of KSR's spoken errors (2404/3120) were the correct spoken form to another numeral within the same numerical class (e.g., 13 → "fourteen," seven hundred one → "six hundred one"), whereas the remaining 33% were generally numerals from another numerical class (e.g., 8 → "sixty," 7 → "twelve"). These errors were not limited to specific numerals. Over the course of testing KSR was able to say each individual number name several times. In addition, because nearly all of his errors were whole numeral substitutions, his errors cannot be attributed to subsequent speech production processes such as retaining individual phonemes in a phonological buffer, or the generation of the motor sequences required to generate individual phonemes.

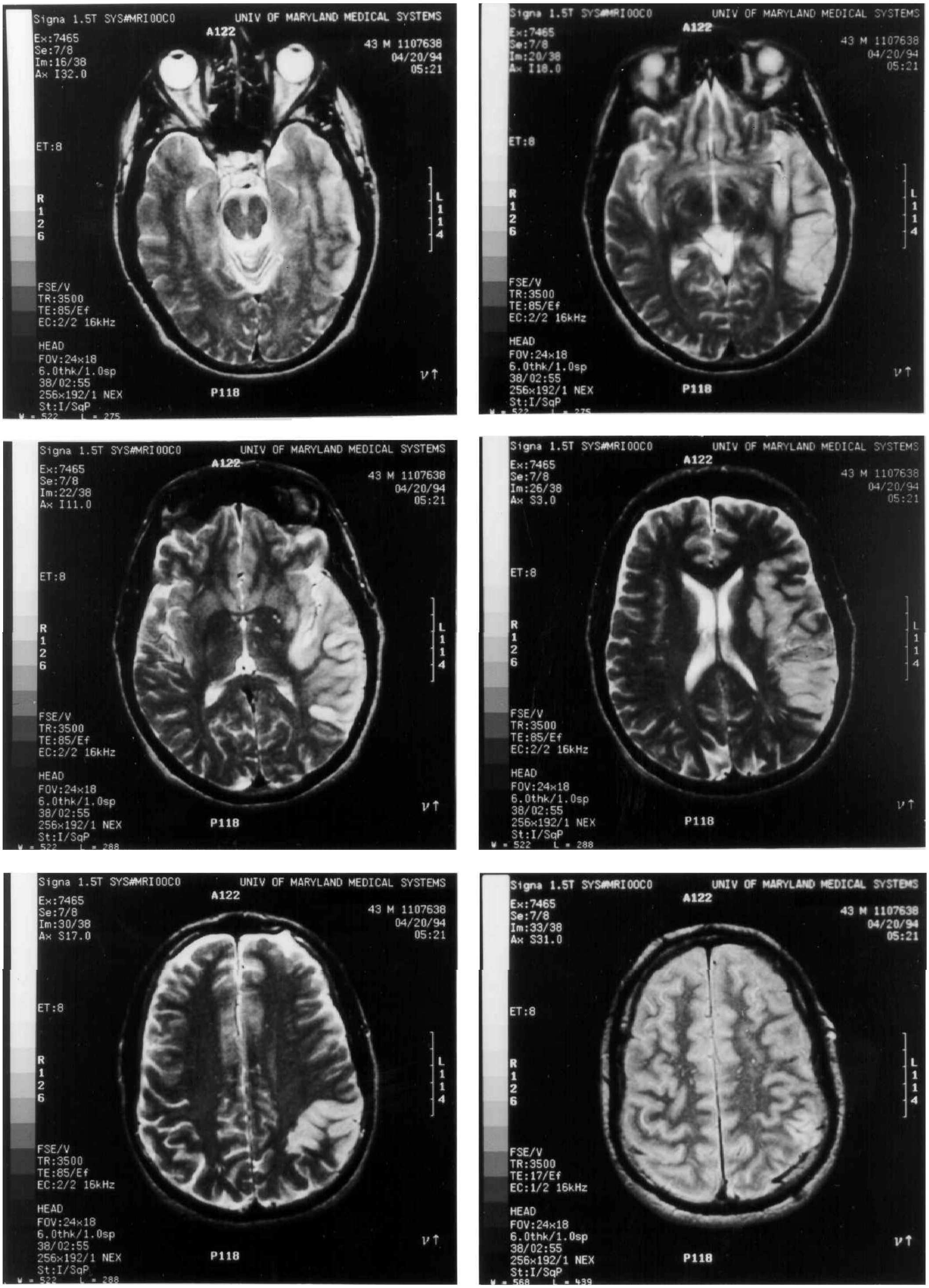


Figure 2. *MRI revealing the locus of KSR's brain injury.*

Arithmetic fact retrieval. We aim to study how arithmetic facts are stored in memory. Therefore, a crucial step in our testing is to determine if KSR has retained the ability to retrieve arithmetic facts from memory. KSR's ability to solve single- and multi-digit addition, subtraction, and multiplication problems quickly and without error (60/60) suggests that he can retrieve arithmetic facts from memory. Additional evidence from timed single-digit arithmetic supports this conclusion. Problems were presented in Arabic form, and responses were typed on a numerical keypad. Collapsing over two runs of the 390 simple arithmetic problems from all four operations, KSR correctly answered 92% of problems (719/780) and his reaction-time pattern revealed a problem size effect typical of that found in healthy adults (Campbell & Graham, 1985). Median reaction times for addition, subtraction, multiplication, and division were 1094, 1136, 1131, and 1358 ms respectively. KSR's RTs are significantly faster than expected if he was solving problems using a nonretrieval strategy such as multiple additions (adding seven eights to solve 7 × 8), which would be expected to take several seconds (LeFevre et al., 1996; Siegler, 1988). KSR's performance pattern, including his speed, accuracy, and characteristic problem size effect, indicate that he is able to retrieve arithmetic facts from memory.

Experiment 1: Evaluation of the phonological storage hypothesis

The phonological storage hypothesis predicts that the arithmetic fact retrieved from memory will reflect the phonological representation of the problem. To evaluate this hypothesis KSR was presented with problems in Arabic form (e.g., 2 × 7) and asked both to say aloud the problem operands ("two times seven") and to write the answer in Arabic form (14). KSR's ability to say the problem aloud was used as an indirect measure of his phonological representation of the problem, and his written answers were used as an indirect measure of his ability to retrieve the appropriate arithmetic fact from memory.

Method

All 390 simple arithmetic table problems (i.e., 0 + 0...9 + 9; 0 – 0...18 – 9; 0 × 0...9 × 9; 0 / 1...81 / 9) were presented one at a time in Arabic form. KSR was asked to say aloud the problem and write the answer in Arabic numerals. Three slightly different versions of this task were performed, each of which resulted in comparable performance and are therefore reported together. The first version of this task required KSR to say the problem aloud and then write the answer. The second version required KSR to write the answer, then say the problem aloud. The third version required KSR to say the problem aloud, say the answer aloud, and then write the answer. Problems were presented briefly, until KSR signalled he was ready to respond (typically less than 1 s). In order to maximise the likelihood that the same representation of the problem was used both to name the problem and retrieve the arithmetic fact from memory, the problems were removed from view before KSR began to respond.

Only errors in saying aloud the problem operands were used to evaluate the phonological storage hypothesis. Operand errors (e.g., + → "times") were not counted because it is possible that the phonological representation of arithmetic facts did not include a representation of the operand.

Results

Over the three tasks using all 390 arithmetic problems, 98% of KSR's written answers for the arithmetic problems were correct (1147/1170). However, he was only able to say aloud 12% of the problem operands correctly (141/1170). Examples of spoken errors are provided in Table 1.

There were a total of 941 "critical" trials in which KSR was unable to say aloud the problem operands and the answer to the presented problem differed

Table 1. *Examples of KSR's spoken and written responses to arithmetic problems*

Problem	*Spoken problem*	*Written answer*
8 + 5	"two plus four"	13
7 – 4	"sixteen minus four"	3
1 × 7	"six times nine"	7
14 / 2	"fifteen divided by five"	7

from the answer to KSR's spoken naming of the problem (e.g., problem: 4 × 6, spoken naming of problem: "three times seven"). For all four operations KSR's written answers to these critical trials were consistent with the problem presented in Arabic form, *not* his naming of the problem operands (e.g., problem: 4 × 6, spoken naming of problem: "three times seven," written answer: 24). Ninety-eight per cent (928/941) of his written answers were correct answers to the original operands, and only one written answer (1/941) correctly answered the problem KSR said aloud (problem: 12 – 4, spoken naming of operands: "fifteen minus six," written answer: 9).

If we assume that KSR's spoken responses reflect his phonological representations of the problem, this pattern of performance strongly contradicts the phonological storage hypothesis. However, before drawing conclusions we will consider the possibility that some spoken responses (which are an indirect measure of the phonological forms used for arithmetic fact retrieval) may be unrepresentative of the actual phonological form hypothesised to be involved in arithmetic fact retrieval.

We consider three types of spoken errors that may be unrepresentative of the underlying phonological form, discussed in detail next: (1) speech production errors, (2) minor phonological representation errors, and (3) perseverations and/or anticipations. In order to provide a highly conservative evaluation of the phonological storage hypothesis, any spoken errors that could *possibly* be attributed to one of the causes given were removed from the final corpus of data used to evaluate the phonological storage hypothesis.

Speech production errors and minor phonological representation errors. A spoken error might be made such that one portion of the problem is incorrect (e.g., "seven times *nim̲e*"). This type of error may be unrepresentative of the underlying phonological representation because an error late in speech production (such as a phoneme substitution) might not reflect a potentially correct phonological representation of the problem.

This type of spoken error could also arise due to a minor error in the phonological representation of the problem (e.g., 5 × 6 → /fr̲aiv taimz sɪks/). In this case, the correct arithmetic fact might still be retrieved because it is sufficiently close to the correct representation (/fr̲aiv/ vs. /faiv/). It was assumed that when the phonological form is close to correct, the correct arithmetic fact would still be retrieved. Since this type of error is *potentially* unrepresentative of the phonological problem representation used to address arithmetic facts, any such spoken error was removed from the analysis of the phonological storage hypothesis. Twelve trials considered to be either speech production errors or minor phonological representation errors were removed from the final data set.

Perseverations and anticipations. Even if KSR said the incorrect number word (e.g., stimulus: 4 × 7; response: "four times *n̲i̲n̲e̲*") he may nevertheless have correctly represented the problem in phonological form (/fɔr taimz sɛvən/). Some brain-damaged patients exhibit a pattern in which they sometimes repeat previous responses (perseverations), or produce upcoming responses (anticipations: e.g., say the answer to a problem when attempting to name the operands). The mechanisms underlying anticipations and perseverations are not well understood (see Cohen & Dehaene, 1998), so it is conceivable that such an error might occur after the generation of the (correct) phonological problem representation, as depicted in Figure 3. In this case the spoken

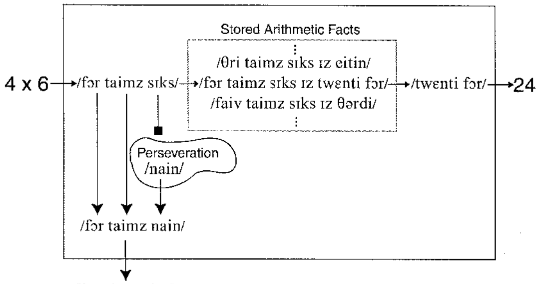


Figure 3. *Implications of a perseverative error for arithmetic fact retrieval.*

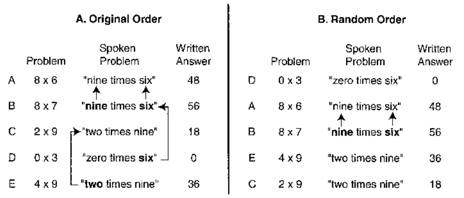
response *might* not be an accurate reflection of the phonological form used to address the arithmetic fact. In order to be highly conservative, *any* such error was removed from the data set used to evaluate the phonological storage hypothesis.

Six trials were removed from the analysis of the phonological storage hypothesis because they were *possible* anticipation errors (e.g., stimulus: 2 + 3; spoken response: "two plus *five*").

Perseverations are more difficult to detect. By definition, perseverations are occasional erroneous spoken errors resulting from intrusions of the phonological representation of previous responses or stimuli. Thus we should determine if KSR's spoken responses sometimes correspond to previous responses or stimuli[1]. However, even if KSR's spoken errors were not at all perseverative, they might still occasionally correspond to previous responses or stimuli simply by chance (because there are so few number words and they are frequently repeated). To determine if KSR was sometimes perseverating, we must first determine how often spoken errors should correspond to previous stimuli and responses simply by chance. If the number of correspondences between spoken errors and previous responses or stimuli exceeds the chance value, it would indicate that some perseverative errors exist in the data corpus.

To calculate the frequency of chance correspondences between KSR's spoken errors and previous responses, the sequence of trials was randomly reordered several times and re-examined for spoken error/previous trial correspondences. The number of chance correspondences between previous trials and spoken errors should not vary between the original order, and the randomly reordered data sets, because in both cases the correspondences are only chance occurrences.

An example of the analysis to be performed is given in Figure 4. As depicted, there are four error/previous trial correspondences in KSR's actual data set (Figure 4a), but only two after the data was randomly reordered (Figure 4b). The difference in error correspondences suggests that there may have been some perseverative responses in the data.



A. Original Order

	Problem	Spoken Problem	Written Answer
A	8 x 6	"nine times six"	48
B	8 x 7	"**nine** times **six**"	56
C	2 x 9	"two times nine"	18
D	0 x 3	"zero times **six**"	0
E	4 x 9	"**two** times nine"	36

B. Random Order

	Problem	Spoken Problem	Written Answer
D	0 x 3	"zero times six"	0
A	8 x 6	"nine times six"	48
B	8 x 7	"**nine** times **six**"	56
E	4 x 9	"two times nine"	36
C	2 x 9	"two times nine"	18

Figure 4. *Comparison of error/previous occurrence correspondences in original and reordered data sets. More correspondences in the original data set provide evidence of perserverative responses.*

There is one more level of complexity to the analysis. In addition to determining whether or not KSR's responses may have been perseverative, we may also ask *how many* previous trials were influencing his responses. Perseverations might involve immediately preceding trials (trial N−1), or the last two preceding trials (trials N−1 and N−2), or more. For this reason different levels of perseveration "depth" were considered separately. If perseverations are found at a depth of 1 (trial N−1), then we considered the level beyond that (i.e., trial N−2), and so on.

Results of perseveration analysis. KSR's data were reordered and reanalysed 1000 times to provide a measure of the likelihood that errors might correspond to a previous response by chance. Table 2 presents the average number of correspondences between errors and previous responses, separated by operation and perseveration depth. For example, the first cell of Table 2 presents the average number of spoken errors in the actual data that corresponded to a stimulus or response found in the immediately preceding trial (N−1; correspondence rate = 18.0). A correspondence rate of 18.0 means that on average 18 errors per hundred trials corresponded to a stimulus or response in an immediately preceding trial. The second column displays the correspondence rates in the randomly reordered data.

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[1] The perseveration analysis sought to determine if an error corresponded with one of several different parts of previous trials, including the actual problem operands, the spoken problem operands, the correct answer, and the written answer.

Table 2. *KSR's average error/previous response correspondence rate relative to chance for addition, subtraction, multiplication, and division*

	Addition			*Subtraction*		
Depth	*Actual*	*Reordered*	*Diff*	*Actual*	*Reordered*	*Diff*
1	18.0	13.2	+4.8	15.7	11.4	+4.3
2	5.0	6.1	−0.9	5.0	5.9	−0.9
3	1.7	2.8	−1.1	2.3	2.9	−0.6
4	1.3	1.5	−0.2	0.3	1.5	−1.2
5	0.3	0.9	−0.6	0.7	0.9	−0.2
	Multiplication			*Division*		
Depth	*Actual*	*Reordered*	*Diff*	*Actual*	*Reordered*	*Diff*
1	17.0	13.0	+4.0	16.0	11.5	+4.5
2	8.0	6.3	+1.7	9	5.9	+3.1
3	1.7	3.1	−1.4	1.7	3.2	−1.5
4	2.3	1.7	+0.6	1.3	1.9	−0.6
5	0.0	1.0	−1.0	0.0	1.2	−1.2

The correspondence rate for KSR's actual data set was slightly higher than that in the randomised sets for the two immediately preceding trials (i.e., N−1 and N−2). For example, KSR had 18 errors that corresponded to stimuli or responses from immediately preceding trials (a depth of 1) in the addition task, whereas the randomly reordered set produced an average of 13.2 error/previous-trial correspondences. This suggests that about 5 of every 18 error correspondences involving immediately preceding trials may be perseverations[2]. Even though only a fraction of such errors are likely to be perseverations, we will provide the phonological storage hypothesis the greatest benefit of the doubt by removing *any possible* perseveration from the analysis, even though most of these errors are unlikely to have arisen from perseverative origins.

All trials in which an erroneous response corresponded to a response, or a stimulus from either of the two immediately preceding trials (trial N−1 and N−2), were removed from the analysis of the phonological storage hypothesis. In total 674 trials were removed, leaving a total of 478 trials for final analysis.

Experiment 1 results and discussion. The final data set should provide a reasonable reflection of KSR's phonological problem representations. We can use this data set to pose our original empirical question: Is the phonological form of the problem used to address arithmetic facts form memory? The answer is clearly "No." KSR produced the correct written answer to the original problem for 100% (132/132) of the trials when the phonological form of the problem was correct produced, *and* for 98% (342/346) of trials when the phonological form of the problem was incorrect. In fact, KSR *only once* retrieved an arithmetic fact that corresponded to the error he made in representing the problem phonologically. This pattern was consistent across all four operations. Clearly, the predictions of the phonological storage hypothesis have been strongly contradicted. KSR's performance indicates that he was not using a phonological representation of the problem to address arithmetic facts in memory.

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[2] The actual data appears to have fewer correspondences at larger perseveration depths (3 to 5). This effect can be attributed to the fact that many more errors in the actual data were scored as smaller perseveration depths (1 and 2), leaving fewer errors to be associated with trials having greater perseveration depths.

Experiment 2: Can KSR rhyme?

A rhyming task was used as a supplementary measure to determine if KSR might have correct phonological numeral representations that are unavailable for spoken production. KSR judged if a word and an Arabic numeral rhymed (e.g., 4 – pour), or did not rhyme (4 – sour). Half of the ryhming and nonrhyming trials had word stimuli that were orthographically similar to the spelling of the Arabic numeral presented (e.g., 4 – pour), and half did not (e.g., 4 – war).

KSR was visually presented examples of words and pictures that rhyme, and was asked to make similar judgements for visually presented word/Arabic-numeral pairs. His number rhyming performance followed the orthographic similarity of the numeral and word (even though the numerals were presented in Arabic form). Almost all pairs that were orthographically similar were judged to rhyme (24/28 judged to rhyme; but only 14/28 correct, whereas approximately half of the non-orthographically similar pairs were judged to rhyme (15/28 judged to rhyme; 17/28 correct). Overall, KSR was correct for only 55% (31/56) of trials in a forced-choice paradigm, revealing no signs of an intact ability to convert stimulus numerals into their appropriate internal phonological form. In sum, we found no evidence that KSR was able to represent numerals accurately in a phonological form.

Experiment 3: Spoken answers and the phonological storage hypothesis

Whereas the previous analysis evaluated the hypothesis that a phonological form of the problem must be used to *address* the arithmetic facts in memory, this section evaluates the notion that answers are retrieved from memory in a phonological form. The phonological storage hypothesis proposes that all answers are *retrieved* in a phonological form and then converted into the form required for production (e.g., written numerals), as depicted in Figure 5.

According to the phonological storage hypothesis, if KSR correctly produces the answer in written form (e.g., stimulus: 5 × 6; response: 30) then the answer must have been correctly retrieved in a phonological form (as shown in Figure 5). This phonological form should be available for both spoken production and for conversion into another form (e.g., Arabic numerals or written number words). Therefore the phonological storage hypothesis predicts that all written and spoken responses should reflect the phonological representation of the answer retrieved from memory (provided there are no speech production errors or perseverations).

Figure 5. *Prediction of the phonological storage hypothesis: Written and spoken answers are based on an arithmetic fact retrieved in phonological form.*

This hypothesis was evaluated by comparing KSR's spoken and written answers to arithmetic problems. Since spoken responses are an indirect measure of KSR's phonological representation of the answer, we evaluate the possibility that some errors in spoken responses may not reflect the underlying phonological form retrieved from memory (e.g., perseverations, or speech production errors).

Method

All 390 simple arithmetic problems were presented in Arabic form blocked by operation. KSR performed two versions of this task, one in which he said the answer and then wrote the answer, and one in which he said the problem, said the answer, and wrote the answer in Arabic form. All responses were produced after the problem was covered to avoid re-encoding of the problem after one response was made.

Results

KSR's written answer performance continued to be highly accurate (99% correct; 774/780), whereas only 35% of his spoken answers were correct (274/780). Table 3 provides examples of KSR's spoken and written answers to the arithmetic problems.

Table 3. *Examples of KSR's spoken and written responses to simple arithmetic problems*

Problem	*Spoken problem*	*Written answer*
7 + 7	"twelve"	14
10 − 1	"eight"	9
7 × 5	"twenty-six"	35
16 / 2	"eight"	4

This pattern of performance appears to contradict the predictions of the phonological storage hypothesis. However, we must ensure that KSR's spoken errors accurately reflect his phonological representation of the answer. For this reason we consider the possibility that some spoken errors reflect either perseverations or speech production errors.

Speech production errors. A total of 15 possible speech production errors (e.g., 16 → "*slixteen*") were identified and removed from the final analysis.

Perseverations. As in the previous experiment, errors corresponding to stimuli or responses in the two preceding trials were considered to be possible perseverations and were removed from the final data set. A total of 323 trials were removed from the analysis. No anticipations were present since there were no responses after the answer was provided.

Overall results and discussion

The remaining 442 trials should reasonably reflect KSR's phonological representation of the answer, and can be used to evaluate the phonological storage hypothesis' prediction: KSR should never be able to produce the correct written answer if the answer is not correctly represented in phonological form. This prediction is clearly contradicted. On 167 of 442 trials KSR was able to write the correct answer despite being unable to say the answer correctly. This pattern held for all four mathematical operations. In fact, KSR was able to produce the correct written answer for 98% of trials whether or not his spoken answer was correct (267/273) or incorrect (167/169). This indicates that KSR was not basing his written answers on an arithmetic fact retrieved in phonological form. In summary, KSR's overall pattern of performance suggests that arithmetic facts are neither stored nor retrieved exclusively in phonological form.

CASE 2: PATIENT JM

Case history

As a consequence of multiple neurological incidents, JM suffers right hemiplegia, right hemianopia, and cognitive impairments. Cerebrovascular accidents (CVAs) in 1978, 1984, and 1985 (age 59) resulted in bilateral damage to the frontal and temporal regions, as well as left temporal and occipital regions. JM's cognitive impairments were first apparent following his CVAs in the mid-1980s.

JM completed the 11th grade, and for 36 years he ran his own business as a bakery distributor. In that capacity, he calculated bills and purchase orders by hand. According to his accountant, these computations were highly accurate. JM was tested from April 1992 to July 1993, when he was 67 years old. He suffers significant comprehension and production impairments in both spoken and written language, which were readily apparent during testing and documented in his medical history.

Arabic numerical comprehension. JM's ability to comprehend Arabic numerals appeared intact. He correctly converted Arabic numerals (0–99) into a dot representation (200/200) and performed without error both single- and multi-digit Arabic numeral comparison judgements (20/20).

Arabic numeral production. JM's ability to produce Arabic numerals was relatively intact. He was able to produce correctly the Arabic numeral that corresponded to a number represented in dots for 91% (182/200) of trials in the range of 0 to 99, and wrote the correct answer to 89% (356/400) of single-digit addition problems (0 + 0 through 9 + 9) presented in Arabic format.

Spoken numeral production. JM had significant impairments in spoken numeral production. When asked to say Arabic numerals aloud, JM correctly produced only 70% (12/20) of numerals in the range of 0–9, and 71% (71/100) of numerals in the range of 0–99. Errors typically involved producing the wrong numeral (e.g., 4 → "eight," 7 → "nine").

Arithmetic fact retrieval. In each of four test sessions, JM performed timed single-digit addition. All 100 addition problems were presented in Arabic form, and JM wrote the answer in Arabic numerals as quickly as possible. Before each trial, he poised his hand as if ready to write. Reaction times were taken by measuring with a stopwatch the time JM took to initiate writing the answer after the problem was presented. JM correctly answered 84% of trials (337/400), with an average RT of 1.2 s ($SD = 0.2$ s). These reaction times, compared to those generally observed for normal adults, are faster than those expected if JM were counting to generate his answers.

In fact, JM was unlikely to count to solve addition problems because he has a significant counting impairment. During each session JM was asked to count aloud from 1 to 18. He was typically unable to count correctly up to numbers larger than 3 or 4. Almost all of his counting errors took the form of misordered, deleted, or repeated number names. For example, he was only able to count up to the number "sixteen" correctly 40% of the time. Also, correlations between JM's reaction times and problem-size measures (such as minimum operand and maximum operand) were found to be non-significant and too small to support a counting hypothesis ($r < .1$). Together these data suggest that JM was retrieving arithmetic facts from memory and not using a counting strategy to solve addition problems.

Experiment 1: Evaluation of the phonological storage hypothesis

JM's performance on an addition task was used to evaluate the notion that arithmetic facts are addressed in memory using a phonological representation of the problem.

Method

Over three sessions JM was presented with the 100 single-digit addition problems 1 at a time, and was asked to say aloud the problem and then write the answer in Arabic numerals. The problem remained in view until JM began to say the problem aloud. Spoken responses were used as an indirect measure of the phonological representation of the problem, and written responses were used as a measure of JM's ability to retrieve arithmetic facts from memory.

Results

JM was able to produce the correct written answer for 91% of trials (273/300), but was only able to say aloud the problem operands correctly for 50% of trials (150/300)[3]. Of the 150 cases in which the spoken operands were incorrect, JM's written answers were consistent with the written form of the problem (134/140) and not with the spoken naming of the problem (2/140). Although this pattern appears to contradict the phonological storage hypothesis, we must ensure that JM's spoken responses accurately reflect his ability to represent the problem in phonological form. Spoken errors that could possibly be unrepresentative of the underlying phonological representation (e.g., anticipations, perseverations, minor phonological encoding errors) were removed from the final data set, as described earlier for patient KSR.

Minor phonological representation or speech production errors. Errors in spoken responses that could possibly be attributed to minor phonological representation errors or speech production (e.g., "*twenty-flive*") accounted for 38 errors. All were removed from the final data set.

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[3] Additionally, for JM we must also consider several other spoken error types, such as operand reversals (e.g., saying "three plus five" for 5 + 3), failures to say aloud correctly (or at all) the operator (e.g., saying "one times seven" for 1 + 7), and failures to maintain correct syntax of a problem's constituents (e.g., "plus four four" for 4 + 4). Even though all technically incorrect, these phonological representations might still retrieve the appropriate arithmetic fact from memory. Therefore, these responses were not used to evaluate the phonological storage hypothesis and were counted as correct spoken responses.

Perseverations and anticipations. JM's error/previous-trial correspondence rate was greater than chance only when considering immediately preceding trials ($N - 1$). Of the corpus of errors, 36 could possibly be considered perseverative, and were removed from the final corpus. Additionally, two spoken errors were considered to be possible anticipations and were removed from the final data set.

Final results

The remaining 224 trials should reasonably reflect JM's phonological problem representations, and can be used to evaluate the phonological storage hypothesis. Of the 62 critical trials in which JM named the problem operands incorrectly, on 58 he produced the correct answer to the problem presented in arabic form, whereas on only 2/62 did he produce the correct answer relative to his naming of the problem. This pattern of performance indicates that JM was not using a phonological representation of the problem to address addition facts from memory.

Experiment 2: Spoken problems and spoken answers to addition problems

JM was also asked to say the problem aloud and say the answer aloud. Methods were otherwise unchanged. The naming of the problem was used as an indirect measure of the phonological representation of the problem, and the spoken answer was used as an indirect measure of the arithmetic fact which was retrieved. The predictions of the phonological storage hypothesis remain the same: JM should only be able to retrieve the correct answer from memory if the phonological representation of the problem was correct.

Results

JM named the problem aloud correctly for only 63% of trials (125/200), but produced the correct spoken answer for 76% (152/200) of trials. Of the 75 critical trials where JM was unable to produce the correct spoken answer, he nevertheless produced the correct written answer 51 times, and only once produced a written answer consistent with his incorrect spoken answer.

After removal of errors that might be attributed to speech production, minor phonological conversion errors, anticipations, or perseverations, 24 trials remained that contradicted the predictions of the phonological storage hypothesis. On these trials, JM was able to retrieve the correct arithmetic fact from memory despite being unable to represent the problem correctly in phonological form.

JM's overall pattern of performance provides additional evidence that arithmetic facts are not addressed in memory by a phonological representation of the problem.

GENERAL DISCUSSION

Taken together, these results strongly contradict the predictions of the phonological storage hypothesis in which the problem and answer are stored in a purely phonological representation. Arithmetic facts are not addressed exclusively in a phonological form (Kolers, 1968; Shanon, 1984). Both patient KSR and JM were often able to retrieve the appropriate arithmetic fact from memory when the problem was not correctly represented in phonological form. The phonological storage hypothesis also proposes that arithmetic facts are retrieved from memory in a phonological form. However, KSR was able to produce the correct written answer even when his phonological representation of the answer was incorrect. Although these results are highly problematic for the phonological storage hypothesis, other theoretical positions appear capable of handling these findings.

Alternative hypotheses

Dehaene's revision of the phonological storage hypothesis. Dehaene modified the triple-code theory to accommodate the findings from brain-damaged patient KSR (Dehaene & Cohen, 1995). The modified theory proposes that arithmetic facts might still be stored and retrieved using a language-based representation, but at the level of representation in which there is a plan for verbal numeral production prior to the insertion of phonological forms (such as Levelt's "lemma," see Levelt, 1989). The performance of KSR and JM would then be interpreted as evidence that they were able to produce the appropriate plan for the production of words composing the verbal numeral, but were unable to fill those slots with the appropriate phonological forms. One consideration is that approximately one third of KSR's spoken errors (716/3120) included errors in selecting the appropriate numerical class (e.g., problem: 7 × 8, spoken operands: "sixteen times four", written response: 56). The final data set used to evaluate the phonological storage hypothesis included 58/346 trials in which KSR's spoken error was outside the correct numerical class. Therefore the triple-code theory must posit that, despite an intact word-frame representation, there are errors both in selecting the phonological stack (tens, teens, ones) and within-stack positions (1, 2, 3…). If the lemma level word-frame representation is unable to constrain the phonological stack selection, as other theories would suggest (McCloskey et al., 1986), it may prove difficult to disambiguate the performance of lemma level representation from an entirely format-independent representation.

Semantic fact retrieval. McCloskey and colleagues propose that arithmetic facts are addressed and retrieved in an abstract-meaning-based form (McCloskey & Macaruso, 1995). This position appears compatible with the results from our subjects. Within this framework, KSR and JM's impairments in representing numerals phonologically should not directly affect the arithmetic fact retrieval process. Both KSR and JM revealed relatively intact abilities to comprehend and produce Arabic numerals. These patients should be able to convert the problem from an Arabic form to an abstract form, which can then be used to address the appropriate arithmetic fact in memory (retrieved in an abstract semantic form), and converted into the appropriate form for Arabic numeral production. An impairment in representing numerals in phonological form would only impair any tasks involving spoken numeral production (such as saying the problems and answers aloud), consistent with the findings of this study. To accommodate other patients who retrieve arithmetic facts that are consistent with their verbal errors (despite evidence of intact Arabic numeral comprehension), such a theory must propose that these patients are recoding the stimuli based on their spoken responses prior to performing arithmetic fact retrieval.

The multi-format position. Campbell's (1994) hypothesis that arithmetic facts are stored in several different codes can account for the results of this study. This study suggests that arithmetic facts are not stored *exclusively* in a phonological form. It is possible that arithmetic facts could be stored in multiple forms, including a phonological form. Our findings cannot address the possibility that normal individuals have several format-specific representations of arithmetic facts, including phonological representations (provided one of the non-phonological representations was unimpaired and available to both patients). This possibility remains a question for future research.

The preferred-code hypothesis. Noel and Seron (1993) propose that numerical skills may be accomplished using a variety of formats, but that we may have preferred codes for handling specific skills, such as retrieving facts from memory. Although this hypothesis has not been fully developed, it is possible that KSR and JM's preferred entry code for access to arithmetic facts was through an Arabic form of representation, rather than a phonologically based representation. Using this framework, arithmetic facts could potentially be represented in a sound-based format, but this seems particularly unlikely in the case of KSR and JM, both of whom exhibit severe phonological production impairments, and are almost exclusively able to retrieve arithmetic facts using Arabic notation. An alternative consistent with our findings is to suggest that these patients are not retrieving language-based arithmetic facts, consistent with Noel and colleagues' arguments that the evidence in support of language/arithmetic interactions have serious weaknesses (Brysbaert et al., 1998; Noel & Seron, 1992; Noel et al., 1997, 1998).

Summary. In summary, we conclude that the available evidence reveals that arithmetic facts are not stored exclusively in a purely phonological form.

Further investigation of theoretical alternatives, such as language-based lemma level representations of multiplication facts, abstract arithmetic fact representations, or multiple-format specific fact representations, deserve further investigation.

Manuscript received 10 July 2001
Revised manuscript received 24 May 2002
Revised manuscript accepted 29 May 2002

REFERENCES

Blankenberger, S., & Vorberg, D. (1997). The single-format assumption in arithmetic fact retrieval. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *23*, 721–738.

Brysbaert, M., Fias, W., & Noel, M.-P. (1998). The Whorfian hypothesis and numerical cognition: Is “twenty-four” processed in the same way as “four-and-twenty”? *Cognition*, *66*, 51–77.

Campbell, J.I.D. (1999). The surface form × problem size interaction in cognitive arithmetic: Evidence against an encoding locus. *Cognition*, *70*, B25–B33.

Campbell, J.I.D., & Graham, D.J. (1985). Mental multiplication skill: Structure, process, and acquisition. Special Issue: Skill. *Canadian Journal of Psychology*, *39*, 338–366.

Campbell, J.I.D. (1994). Architectures for numerical cognition. *Cognition*, *53*, 1–44.

Campbell, J.I.D. (1997). Reading-based interference in cognitive arithmetic. *Canadian Journal of Experimental Psychology*, *51*, 74–81.

Campbell, J.I.D. (1998). Notation and linguistic influence in cognitive arithmetic: Comment on Noel, Fias, and Brysbaert. *Cognition*, *70*, 25–33.

Chochon, F., Cohen, L., Van de Moortele, P.F., & Dehaene, S. (1999). Differential contributions of the left and right inferior parietal lobules to number processing. *Journal of Cognitive Neuroscience*, *11*, 617–630.

Cohen, L., & Dehaene, S. (1998). Competition between past and present—Assessment and interpretation of verbal perseverations. *Brain*, *121*, 1641–1651.

Cohen, L., & Dehaene, S. (2000). Calculating without reading: Unsuspected residual abilities in pure alexia. *Cognitive Neuropsychology*, *17*, 563–583.

Cohen, L., Dehaene, S., Chochon, F., Lehericy, S., & Naccache, L. (2000). Language and calculation within the parietal lobe: A combined cognitive, anatomical and fMRI study. *Neuropsychologia*, *38*, 1426–1440.

Dagenbach, D., & McCloskey, M. (1992). The organisation of arithmetic facts in memory: Evidence from a brain-damaged patient. *Brain and Cognition*, *20*, 345–366.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, *44*, 1–42.

Dehaene, S., & Cohen, L. (1991). Two mental calculation systems: A case study of severe acalculia with preserved approximation. *Neuropsychologia*, *29*, 1045–1074.

Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition*, *1*, 83–120.

Dehaene, S., & Cohen, L. (1997). Cerebral pathways for calculation: Double dissociation between rote verbal and quantitative knowledge of arithmetic. *Cortex*, *33*, 219–250.

Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science*, *284*, 970–974.

Delazer, M., & Bartha, L. (2001). Transcoding and calculation in aphasia. *Aphasiology*, *15*, 649–679.

Delazer, M., & Benke, T. (1997). Arithmetic facts without meaning. *Cortex*, *33*, 697–710.

Delazer, M., Girelli, L., Semenza, C., & Denes, G. (1999). Numerical skills and aphasia. *Journal of the International Neuropsychological Society*, *5*, 213–221.

Frenck-Mestre, C.A., & Vaid, J. (1993). Activation of number facts in bilinguals. *Memory and Cognition*, *21*, 809–818.

Fulbright, R.K., Molfese, D.L., Stevens, A.A., Skudlarski, P., Lacadie, C.M., & Gore, J.C. (2000). Cerebral activation during multiplication: A functional MR imaging study of number processing. *American Journal of Neuroradiology*, *21*, 1048–1054.

Gallistel, C.R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition*, *44*, 43–74.

Girelli, L., & Delazer, M. (1999). Differential effects of verbal paraphasias on calculation. *Brain and Language*, *69*, 361–364.

Girelli, L., Delazer, M., Semenza, C., & Denes, G. (1996). The representation of arithmetical facts: Evidence from two rehabilitation studies. *Cortex*, *32*, 49–66.

Goodglass, H., & Kaplan, E. (1983). *The assessment of aphasia and related disorders.* Philadelphia, PA: Lea & Febiger.

Hittmair-Delazer, M., Semenza, C., & Denes, G. (1994). Concepts and facts in calculation. *Brain*, *117*, 715–728.

Kolers, P.A. (1968). Bilingualism and information processing. *Scientific American*, *218*, 78–86.

Lampl, Y., Eshel, Y., Gilad, R., & Sarova-Pinhas, I. (1994). Selective acalculia with sparing of the subtraction process in a patient with left parietotemporal haemorrhage. *Neurology*, *44*, 1759–1761.

LeFevre, J.-A., Bisanz, J., Daley, K.E., Buffone, L., Greenham, S.L., & Sadesky, G.S. (1996). Multiple routes to solution of single-digit multiplication problems. *Journal of Experimental Psychology: General*, *125*, 284–306.

LeFevre, J.-A., Qingwen, L., Smith-Chant, B.L., & Mullins, D.B. (2001). Multiplication by eye and by ear for Chinese-speaking and English-speaking adults. *Canadian Journal of Experimental Psychology*, *54*, 277–284.

Levelt, W.J.M. (1989). *Speaking: From intention to articulation.* Cambridge, MA: MIT Press.

Marsh, L.G., & Maki, R.H. (1976). Efficiency of arithmetic operations in bilinguals as a function of language. *Memory and Cognition*, *4*, 459–464.

McCloskey, M. (1992). Cognitive mechanisms in numerical processing: Evidence from acquired dyscalculia. *Cognition*, *44*, 107–157.

McCloskey, M., Aliminosa, D., & Macaruso, P. (1991). Theory-based assessment of acquired dyscalculia. Special Issue: Cognitive and neuropsychological aspects of calculation disorders. *Brain and Cognition*, *17*, 285–308.

McCloskey, M., & Macaruso, P. (1995). Representing and using numerical information. *American Psychologist*, *50*, 351–363.

McCloskey, M., Sokol, S.M., & Goodman, R.A. (1986). Cognitive processes in verbal-number production: Inferences from the performance of brain-damaged subjects. *Journal of Experimental Psychology: General*, *115*, 307–330.

McNeil, J., & Warrington, E. (1994). A dissociation between addition and subtraction with written calculation. *Neuropsychologia*, *32*, 717–728.

Noel, M.-P., Fias, W., & Brysbaert, M. (1997). About the influence of the presentation format on arithmetical- fact retrieval processes. *Cognition*, *63*, 335–374.

Noel, M.-P., Robert, A., & Brysbaert, M. (1998). Does language really matter when doing arithmetic? Reply to Campbell (1998). *Cognition*, *67*, 365–373.

Noel, M.-P., & Seron, X. (1992). Notational constraints and number processing: A reappraisal of the Gonzales and Kolers (1982) study. *Quarterly Journal of Experimental Psychology*, *45A*, 451–478.

Noel, M.-P., & Seron, X. (1993). Arabic number reading deficit—a single-case study or when 236 is read (2306) and judged superior to 1258. *Cognitive Neuropsychology*, *10*, 317–339.

Pesenti, M., Seron, X., & Vanderlinden, M. (1994). Selective impairment as evidence for mental organization of arithmetical facts—Bb, a case of preserved subtraction. *Cortex*, *30*, 661–671.

Pesenti, M., Thioux, M., Seron, X., & De Volder, A. (2000). Neuroanatomical substrates of Arabic number processing, numerical comparison, and simple addition: A PET study. *Journal of Cognitive Neuroscience*, *12*, 461–479.

Rickard, T.C., Romero, S.G., Basso, G., Wharton, C., Flitman, S., & Grafman, J. (2000). The calculating brain: An fMRI study. *Neuropsychologia*, *38*, 325–335.

Rossor, M.N., Warrington, E.K., & Cipolotti, L. (1995). The isolation of calculation skills. *Journal of Neurology*, *242*, 78–81.

Shanon, B. (1984). The polyglot mismatch and the monolingual tie. Observations regarding the meaning of numbers and the meaning of words. *New Ideas in Psychology*, *2*, 75–79.

Siegler, R.S. (1988). Strategy choice procedures and the development of multiplication skill. *Journal of Experimental Psychology: General*, *117*, 258–275.

Sokol, S.M., McCloskey, M., Cohen, N.J., & Aliminosa, D. (1991). Cognitive representations and processes in arithmetic: Inferences from the performance of brain-damaged patients. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *17*, 355–376.

Spelke, E.S., & Tsivkin, S. (2001). Language and number: A bilingual training study. *Cognition*, *78*, 45–88.

van Harskamp, N.J., & Cipolotti, L. (2001). Selective impairments for addition, subtraction, and multiplication. Implications for the organisation of arithmetical facts. *Cortex*, *37*, 363–388.

Warrington, E.K. (1982). The fractionation of arithmetical skills: A single case study. *Quarterly Journal of Experimental Psychology*, *34A*, 31–51.

APPENDIX A

Patient KSR: Boston Diagnostic Aphasia Exam

Severity Rating: 3

Fluency
Articulation rating: 5
Phrase length: 6
Melodic line: 6
Verbal agility: 9
Auditory comprehension
Word discrimination: 46
Body-part identification: 10
Commands: 11
Complex ideational material: 4
Naming
Responsive naming: 10
Confrontation naming: 28
Animal naming: 3

Severity Rating: 3

Reading
Word reading: 7
Oral sentence reading: 1
Repetition
Repetition of words: 7
High-probability: 5
Low-probability: 0
Paraphasia
Neologistic: 1
Literal: 12
Verbal: 9
Other: 0
Automatic speech
Automatised sequences: 4
Reciting: 0

Severity Rating: 3

Reading comprehension
Symbol discrimination: 8
Word recognition: 6
Comprehension of oral spelling: 1
Word-picture matching: 9
Reading sentences and paragraphs: 2
Writing
Mechanics: 5
Serial writing: 46
Primer level dictation: 4
Spelling to dictation: 2
Written confrontation naming: 7
Sentences to dictation: 0
Narrative writing: 1