Transition to Advanced Mathematics

Version 4.0

Course Prospectus

A research-based mathematics course combining traditional and innovative teaching techniques to help students build conceptual understanding of key mathematical ideas.
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Many students enter high school with weak preparation for Algebra I and negative attitudes towards their own mathematical abilities. Algebra 1 is often called a "gatekeeping course." Students who succeed in Algebra 1 are in a good position to progress to more advanced high school mathematics and science courses, which, in turn, make them good candidates for postsecondary education or technical jobs right after graduation. The sooner a student can master Algebra 1, the sooner they can move on to more advanced classes.

Although many high schools offer some form of remediation in mathematics and reading, these efforts are typically not grounded in a well-developed research base or supported by solid effectiveness evaluations.

What is clear is that the type of accelerated learning required by poorly prepared high school students needs to involve more than narrow test preparation. It must be substantial, sustained, and enable students to rapidly develop declarative, procedural, and meta-cognitive knowledge. It also has to motivate students to learn and take advantage of their strengths.

Transition to Advanced Mathematics (TAM) is a research-based ninth grade mathematics course combining traditional and innovative teaching strategies. TAM encourages students' conceptual understanding of key mathematical ideas that underlie high school mathematics and sharpens their overall basic skills. TAM challenges students to think through and make sense of what they are doing, learn from one another, share and respect ideas, and make connections between mathematics and the real world.

TAM is designed to work in conjunction with Algebra I by offering ninth grade students a year-long “double dose” of mathematics instruction. TAM is designed for 80-90 minute lessons. In most districts, passing algebra is becoming a requirement for promotion into the tenth grade. Successful implementation of TAM occurs in schools that provide multiple layers of support matched with a schedule that allows for double dosing.

Ensuring that all high school students, regardless of background or income level, receive high quality instruction and rigorous coursework in college-preparatory mathematics will result in substantial social and economic benefits at both the individual and national levels. (Betts, 1998; Bishop & Mane, 2001)
Learning Mathematics is the Product of at Least Three Factors.

1. Knowledge and Experience

Over 30 years of cognitive research has reaffirmed the obvious: effective strategies are based on a solid foundation of knowledge. Expert knowledge, in turn, comes from experience, structured learning opportunities, and reflective thought. The same research has also shown that people do not simply learn what they are told. Every new piece of information is interpreted through the lens of both correct and incorrect prior knowledge.

2. Strategic and Active Thought

Real and lasting learning is the product of active and purposeful thought. New information needs to be integrated and connected to our existing knowledge.

3. Desire and Belief

If we believe something is unimportant, we will not learn it. We may attend to it for a short duration if required, but we will not learn it. In general, the usefulness and power of mathematics is not made apparent to students. Also, if we do not believe we have a chance for success, we are unlikely to put forth our best effort for risk of public failure. Students, like most people, are often taught to believe that a talent for mathematics is a gift, which they either do or do not have. They also believe, like most people, that mathematics is a sequential subject. In school this usually boils down to the belief that you must master advanced arithmetic before you can learn other mathematical topics. Neither of these commonly held beliefs is true, yet they often prevent students from risking effort and becoming engaged with mathematics.
Transition Courses Tend to be Successful When They. . .

Create an Environment that Encourages and Supports a Variety of Learning/Thinking Styles.

Most mathematical problems and procedures can be solved and employed visually, verbally, and symbolically with concrete models. Students who struggle with symbolic approaches are very often effective visual, concrete, or verbal problem solvers. Traditionally, however, school mathematics has been designed only to recognize and promote strategies based on symbolic manipulation.

Create an Environment that Makes Effective Use of Peer-Assisted Learning

Research shows that there are effective and ineffective forms of cooperative learning. Unstructured group work usually brings few benefits. There is, however, compelling evidence that peer-assisted learning leads to higher classroom achievement when students work together on a structured task that provokes thought, requires everyone to participate, and provides multiple opportunities for students to share their thinking process.

Create an Environment that Builds on the Mathematical Knowledge and Insights Students Have Acquired In and Out of School and Demonstrates the Relevance of Mathematics to Their Current and Future Lives.

Students who have weak formal arithmetic skills can possess strong mathematical insights. A classroom of students with weak formal arithmetic skills can still provide the necessary prior knowledge and insights which will enable the class to explore interesting and challenging mathematical topics. These topics should be chosen to demonstrate and highlight the power and significance of mathematics to students’ current and future lives.
What Can Teachers Expect From Transition to Advanced Mathematics?

- Consumable student materials that engage students in transitional activities to build a foundation for success in algebra
- Opportunities for peer-assisted learning
- Materials that encourage procedural skills and conceptual understandings of the "big ideas" in mathematics
- Tasks designed to reveal students' pre-conceptions and misconceptions about math
- Strategies to promote student metacognitive and self-directed thinking and learning
- Classrooms where students learn from one another, share ideas, and respect others’ ideas
- Lessons connecting mathematics to the real world
- Instructional strategies that engage students
- Habits of mind development to transition from concrete arithmetic thinking to abstract algebraic thinking
Instructional Strategies

Class Discussions
Lectures
Positive/Negative Attributes
Counting Off
Four Corners
Investigation
Guided Note Taking
Guided Practice
Predicting
Motivating Questions
Outcome Sentences
Practicing the Scientific Method
Projects
Question, Write, Respond
Student Groupwork
Thinking Aloud
Three in a Row
“What” Questions
Pass it Along
TAM
Teacher’s Support System

Before During & After

On-site Coaching
On-site Facilitators

Consumable Materials
Professional Development
Unit 1: Mathematical Reasoning, Data Analysis, and Probability

Lesson 1: Topics in Number Theory
• Develop an intuitive understanding of prime number, composite number, and factor
• Prove a conjecture false by counterexample
• Observe patterns and make generalizations based on their observations

Lesson 2: Number Patterns
• Discover number patterns
• Construct the meaning of divisibility
• Illustrate divisibility rules
• Utilize the KWL strategy
• Elicit prior knowledge about prime numbers
• Make generalizations based on observations

Lesson 3: Inductive Reasoning
• Observe patterns and make generalizations based on observations
• Construct the meaning of inductive reasoning
• Develop logical- and visual-thinking skills
• Understand that faulty inductive reasoning may lead to prejudices and stereotypes

Lesson 4: Data Analysis
• Collect and describe data
• Construct and interpret charts and graphs
• Make inferences and arguments based on data analysis
• Examine arguments based on data others have analyzed

Lesson 5: Descriptive Statistics
• Determine the measures of central tendency
• Understand measures of dispersion
• Make judgments and arguments based on graphs and statistics

Lesson 6: Probability
• Determine the likelihood of an event happening
• Conduct experiments or simulations and examine outcomes
• Develop an appreciation for a simulation approach to solving problems

Unit 2: Integers

Lesson 1: Natural Numbers to Integers
• Students develop an understanding of the organizational structure and relationships between natural numbers, whole numbers, and integers.
• Engage in activities on closure of operations to develop the need for the set of integers
• Investigate the historical development of natural numbers, whole numbers, and integers

Lesson 2: Adding Integers
• Extend understandings of addition of whole numbers to integers
• Engage in activities to develop a rule for adding integers
• Apply the addition of integers to real-world contexts

Lesson 3: Subtracting Integers
• Extend the concept of subtracting whole numbers to subtracting integers
• Engage in activities to develop a rule of subtracting integers
• Apply the addition of integers to real-world contexts
Lesson 4: Multiplying Integers
• Engage in activities to develop an intuitive understanding of the rules for multiplying integers
• Apply multiplication of integers in real-world contexts

Lesson 5: Dividing Integers
• Engage in activities to develop an intuitive understanding of the rules for dividing integers. Division is limited to integer quotients.

Lesson 6: From Integers to Real Numbers
• Students develop an understanding of the organizational structure and relationships between natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers.
• Engage in activities on closure of operations to develop the need for the set of rational numbers and irrational numbers
• Students investigate the historical development of rational and irrational numbers.

Unit 3: Rational Numbers
Lesson 1: Relative and Absolute Thinking
• Understand the difference between absolute thinking and relative thinking
• Approach a problem by thinking relatively
• Help students rethink their thinking about rational numbers

Lesson 2: Percents
• Engage prior knowledge about percents
• Encourage multiplicative reasoning through halving and splitting
• Introduce the concept of a unit when taking percent

Lesson 3: Finding Percents
• Students will conceptually link taking the percent of a number with partitioning.
• Students will conceptually understand what taking a percent of a number means.
• Students will be able to estimate the percent of a number to solve real-world problems.

Lesson 4: The Multiple Personalities of Rational Numbers and Unitizing Revisited
• Students will review the different meanings of a/b.
• Students will use the part/whole representation to review representing a relationship with a fraction and change the representation depending on the unit.

Lesson 5: Common Denominators and Adding/Subtracting Fractions
• Students will use prior knowledge to understand the purpose for common denominators.
• Students will use the multiplicative identity to help with adding and subtracting fractions.

Lesson 6: Multiplying Rational Numbers
• Students will discover or demystify the algorithm for multiplication.
• Students will compare how finding the percent of a number is the same as multiplying by a fraction.
(Optional Enrichment)

Lesson 7: Reciprocals and Dividing Fractions
• Students will conceptually understand the definition of the reciprocal.
• Students will use the reciprocal to simplify division expressions with rational numbers.

Lesson 8: Investigating the Mysteries of Decimals
• The students will be able to complete the basic operations of addition, subtraction, and multiplication of decimal numbers in an application and as a procedure.

Lesson 9: Ratios and Rates
• Students will be able to compare ratios and rates and solve problems using ratios and rates.
• Students will organize their understanding of rational numbers in a graphic organizer.
Unit 4: Measurement
Lesson 1: Measuring Concepts and Skills
• Gain a firm understanding of how to measure and what it means to measure.
• Use informal and standard units to measure.
• Understand measuring instruments.

Lesson 2: Areas and Perimeters
• Construct the meaning of area and perimeter of a figure
• Understand that perimeter is the length around the edge of the figure
• Understand the idea that area is a measuring of covering

Lesson 3: Rectangles & Parallelograms
• Students will discover formulas to find the areas of rectangles and parallelograms.
• Students will develop inductive reasoning, problem solving, and cooperative behavior skills.

Lesson 4: Angles and Angle Measure
• Construct the meaning of angles and how to measure them.
• Classify angles.
• Define and classify triangles.
• Identify the initial and terminal sides of an angle
• Identify positive and negative angle measures

Lesson 5: Getting Ready for Pythagoras
• Discover the Pythagorean Theorem Formula
• Practice using geometric tools
• Learn new vocabulary
• Develop inductive reasoning, problem solving, and cooperative behavior

Lesson 6: Locations for Real Numbers
• Describe locations of points on grids and number lines
• Identify locations of points on grids and number lines
• Create coordinate systems
• Practice construction skills

Lesson 7: Slope
• Develop a clear understanding of the slope of a line
• Discover a formula for finding a slope of a line in a coordinate plane
• Demonstrate how a slope can be represented by a quantity in a formula and by a graph of a line
• Reinforce global coordinates using latitude and longitude

Lesson 8: Slopes for Special Lines
• Draw a line given its slope and a point on the line.
• Construct the meaning of slopes of parallel and perpendicular lines.

Unit 5: Patterns and Functions – Introduction to Algebra
Lesson 1: Order of Operations
• Know the order of operations
• Simplify expressions using the order of operations
• Use the order of operations to evaluate real-world expressions
• Decimal place value review
• Exponent review
• Writing numbers in scientific notation

Lesson 2: Equivalence
• Create equivalent expressions
• Create equivalent equations

Lesson 3: Opposite Operations
• Complete the opposite operation
• Recognize the operation in algebra notation
Lesson 4: Solving One-Step Equations
- Students solve equations using opposite operations.
- Check a solution to an equation
- Translate real-world problems into equations and solve them using opposite operations

Lesson 5: Solving Two-Step Equations
- Students solve two-step equations using opposite operations, algebra tiles, and algebraic symbols
- Check a solution to an equation
- Translate application problems into equations and solve

Lesson 6: Relations and Data Tables
- Describe relationships between sets of data
- Draw and interpret graphs from relations and data tables
- Identify independent and dependent variables
- Analyze real-world data
- Obtain relations from graphs and data tables

Lesson 7: Patterns
- Interpret relationships between sets of data
- Analyze real-world data
- Express mathematical ideas graphically, orally, and in writing
- Develop critical thinking and cooperative behavior

Lesson 8: Introduction to Functions
- Interpret relationships between quantities
- Analyze real-world data
- Express mathematical ideas graphically, orally, and in writing
- Develop critical thinking and cooperative behavior
- Learn domain, range, function, and function notation

Lesson 9: Exploring Equations
- Construct the meaning of equations as another form of representing functions
- Use at least five different forms of representing a function
What Does a TDHS Mathematics Classroom Look Like?

The National Council of Teachers of Mathematics has recommended principles and standards for school mathematics. These principles guide what a TDHS classroom should look like at highest levels of implementation.

**Equity**

*High expectations and strong support for all students.*

What does it look like?

- Teachers communicating high expectations in interactions during classroom instructions.
- Thoughtful assignment of cooperative learning groups and facilitation of those groups.
- Feedback on assignments.
- Providing technology to give all students opportunities to participate in activities with challenging mathematics.
- Skillful use of learning stations to support differentiating instruction for diverse learners.

**Curriculum**

*A coherent curriculum focused on important mathematics.*

What does it look like?

- TDHS provides a strong, focused, coherent curriculum during the first semester.
- Lessons should include a "bellringer"/"warm up" — TDHS employs the "Problems of the Day." We recommend they are used as peer assisted problems where students work with a partner.
- Students should enter the classroom and be immediately engaged in the “Problem of the Day” activity without direction.
- Lessons provide a "Setting the Stage" which is a whole-group advanced organizer for the direct instruction component of the lesson.
- Using the student journal activities, students work in cooperative groups and are actively engaged in guided practice. Usually followed by small group presentations and individual exercises.
- In the second semester, the coaches work with the teachers to infuse the same strategies in your district curricula.
**TEACHING**
*Understanding what students know and need to learn and then challenging and supporting them to learn it well.*

What does it look like?

- Questioning techniques that reveal students' prior knowledge and build on this knowledge.
- TDHS teachers use curricular materials provided and engage in reflective practice with the site based coaches and peers and instructional facilitators.
- TDHS teachers create an environment of student discussion and collaboration.
- Because teaching requires continual improvement, teachers are strongly encouraged to participate in ongoing professional development opportunities offered throughout the year.

**LEARNING**
*Students must learn mathematics with understanding actively building new knowledge from prior knowledge and experience.*

What does it look like?

- TDHS Mathematics classrooms have students who are actively engaged in presented mathematical tasks and involved in classroom interactions.
- You will see social interaction and classroom discourse around mathematical reasoning. This will often include students working in cooperative groups, making presentations and sharing their reasoning about solutions.

**ASSESSMENT**
*Assessment should support the learning of important mathematics and furnish useful information to teachers and students.*

What does it look like?

- TDHS Mathematics teachers use a variety of assessment techniques. You will see evidence of students' progress toward standards in tests and quizzes that include open ended questions, brief and extended constructed tasks, menu driven formal assessments, as well as in conversations, journals and portfolios.

**TECHNOLOGY**
*Technology is essential in teaching and learning mathematics; it influences the mathematics taught and enhances student learning.*

What does it look like?

- TDHS Mathematics teachers are strongly encouraged to use technology to help students develop skills and procedures to better understand mathematics.
- Calculators and computers allow students to examine examples and representational forms of mathematical ideas. You will often see graphic calculators used routinely, stations of computers for individualized or small group rotations, spreadsheets and dynamic geometry software where possible.
What Research, Teachers, and Students Have to Say About TAM

Transition to Advanced Mathematics is currently being taught in 12 states. Our researchers and curriculum development teams rely heavily on feedback from teachers and students alike. Here are some of the research results and comments made by teachers and students regarding their experiences with TAM.

Our best research evidence on the effectiveness of the TAM/Algebra 1 sequence comes from a randomized study in Baltimore. Students and teachers in six nonselective Baltimore high schools were randomly assigned to teach either TAM/Algebra 1 or a first-semester mathematics course of their own choosing. Both courses were "double-dose" courses in extended periods. All students took the CTBS test in the fall and spring.

In fall to spring growth, TAM/Algebra 1 students outscored the comparison students by half a year's growth (11 scale score points, and 7 national percentile points).


"[TAM] really helped me cover more Algebra I material. I had a 90% pass rate on the Maryland Functional Math Test."

Mrs. Watts, Patterson High School, Baltimore, MD

Student Surveys

82% of students stated that TAM made them feel more confident in their mathematical abilities.

64% stated that they paid more attention in the TAM class than in other math classes.

75% of Baltimore city students reported: “Because of this class, I understand math better.”

70% of Baltimore city students reported: “Because of this class I feel more confident about my ability to do math.”

69% of students reported they were “learning new strategies, concepts, and skills.”

42% of students reported that TAM motivated them to attend school.

“I have enjoyed watching the students’ ability to discuss mathematical concepts improve dramatically with this program.”

Mrs. Nagy, Southwestern High School, Baltimore, MD

“I love the hands on approach to the TAM curriculum. My students enjoy the real world applications to the problems.”

Susan Smith, Whiteville High School, Whiteville, NC

“I really like how the Transition to Advanced Mathematics material has the students work in groups. The students are more willing to share.”

“I really like how the students have to write explanations, because sometimes their explanations are different than their computations.”

Becky Smith, St. Peter Cristo Rey High School, Omaha, NE
Transition to Advanced Mathematics
Course Materials

Every TAM teacher receives all the components that make the course successful.

Course Consumables

Unit 1: Teacher Manual, Student Journals, Lesson Transparencies
Unit 2: Teacher Manual, Student Journals, Lesson Transparencies
Unit 3: Teacher Manual, Student Journals, Lesson Transparencies
Unit 4: Teacher Manual, Student Journals, Lesson Transparencies
Unit 5: Teacher Manual, Student Journals, Lesson Transparencies

In addition to TAM’s consumable materials, Talent Development High Schools provides each teacher with a “Resource Tub” that contains additional lesson materials, reference, and resource information.

1–Roll of Twine (250 feet)  
1–Package of Assorted Rubber Bands  
2–Wax Paper Rolls  
3–Boxes of Jumbo Paper Clips  
Sticky Notes  
1–Equals Game  
1–Cooperative Learning Book  
1–Elementary & Middle School Mathematics  
15–Measuring Tapes English/Metric  
1–Timer  
30–6” Protractors  
30–Algebra Tile Student Sets  
1–Set Game  
1–Overhead Algebra Tiles (set of 70)  
10 sets of 4 Jars  
30–Safe T-Rulers  
30–Safe Drawing Compasses  
1 Pack Transparency Sheets (100 sheets)  
Coordinate Dry Erase Board Classroom Set (set of 30)  
3–Packs of Overhead Markers  
2–Rolls of Masking Tape (60 yd)  
2–rolls of Scotch Tape  
4–packs of Construction Paper (50 sheets)  
8–packs of Colored Pencils (set of 12)  
1–9x12 Clipboard  
7–packs of Magic Markers (set of 8)  
30–Scissors  
1–pack Spring Clothes Pins (30 pack)  
1–Rough Tote Storage Tub (18 gal)
“Problem of the Day” is a peer-assisted starter activity designed to spark students’ problem solving abilities, develop their number sense, and increase their mathematical communication skills.

Each lesson begins with a “Setting the Stage” exercise. The goal is to engage students in a short review or introduce new topics to help them transition into the lesson. This is generally a whole class activity.

“Discovery Activities” are generally the longest part of a lesson (30-60 minutes) and often use manipulatives to engage students in an activity that ties a concept to a skill. There are a variety of instructional strategies and cooperative learning techniques implemented in these activities.

These problems are generally an extension to the Discovery Activity(s). They are designed to bridge the concept back to the skill. These are best done in pairs with a whole-class discussion following to solidify concepts and skills.

These problems are generally focused on various careers or applications of the concepts and skills learned in the lesson and how they can be applied to real-world situations.

One advantage of extended periods is that the 80–90 minute block enables teachers to provide both significant whole class and small group instruction within a single period. Teachers can choose from a array of activities, including:

- Thinking Worksheets
- Learning Stations
- Study Groups
- Peer Tutoring and Computer Assisted Instruction
A Sample TAM Lesson

In the next few pages, we are providing a sample lesson from *Transition to Advanced Mathematics*.

We begin, as a TAM class begins, with two example “Problems of the Day.” These warm up activities are used as peer assisted problems where students work with a partner.

After the “Problems of the Day” is the sample Student Journal. After the Student Journal example is an example of the TAM Teacher’s Manual.
Problem Set 13

**No Hands Math**
Try to do the following problems in your head. Be prepared to explain your answers.

1. Name two time periods that are equivalent to 1 day.
   (For example, 24 hours, \(\frac{1}{7}\) of a week, and so on.)

2. Name two time periods that are equivalent to 7 days.

3. \[
\begin{array}{c}
  435 \\
  \times 24
\end{array}
\]
   a. 2,610
   b. 10,440
   c. 3,513
   What was the first choice you eliminated?
   Why?
   What was the second choice you eliminated?
   Why?

4. \[
\begin{array}{c}
  24)864
\end{array}
\]
   a. 5
   b. 1,589
   c. 36
   What was the first choice you eliminated?
   Why?
   What was the second choice you eliminated?
   Why?
Problem Set 23

No Hands Math
Try to do the following problems in your head. Be prepared to explain your answers.

1. Where will you end up if you start at 0, lose 6, gain 7, and then lose 4? __________

2. Where will you end up if you start at 5, lose 15, gain 5, and then gain 1? __________

3. Where will you end up if you start at 2, lose 3 twice, and then gain 4? __________

Does This Make Sense?

Does the following statement make sense? Explain in writing and to a partner.

Five aliens landed in New Mexico 2 years ago. Last year, each alien gave birth to 3 baby aliens. This year, each of those baby aliens had 4 babies. Assuming none of the aliens died and that no new aliens came to Earth, there are now a total of 12 aliens living in New Mexico.

(Hint: Use this space to draw a picture.)
Unit 1: Mathematical Reasoning, Data Analysis, and Probability
Sample Lesson
Student Journal
Discovery Activity

Graphs and tables give a lot of data. We can summarize that data by producing a few numbers that describe the complete set. These numbers are called statistics. Statistics are used to describe the center of the data and the spread of the data.

Three measures of central tendency are mode, median, and mean.

The mode is the value that occurs most frequently in the set of data. If two or more values are the most frequent, there is more than one mode.

1. Consider Terri’s math scores.

   81, 62, 95, 65, 62, 73, 84, 65, 56, 65, 77, 82, 90

   The mode of this set is __________________________

2. Order Terri’s math scores from least to greatest.

   __________________________________________

   The median is the middle value in an ordered set of data. Half of the values are either above or below the median. If there is an even number of values, the median is the average of the two middle values.

3. The median of this set of Terri’s math scores is __________________________

   The mean is the sum of all the values in the set of data divided by the total number of values.

4. The mean of this set of Terri’s math scores is __________________________

Terri took an additional math test and scored a 100. Terri’s math scores now look as follows:

81, 62, 95, 65, 62, 73, 84, 65, 56, 65, 77, 82, 90, 100

5. The mode of this set is __________________________

6. The median of this set is __________________________

7. The mean of Terri’s math scores is __________________________
8. What does the mean of Terri’s math scores tell us?

___________________________________________________________________________________
___________________________________________________________________________________
___________________________________________________________________________________
___________________________________________________________________________________
___________________________________________________________________________________

Three ways of measuring spread are the maximum, minimum, and range.

The maximum is the greatest number.

The minimum is the least number.

The range is a single number that tells the difference between the maximum and the minimum values.

9. Determine the maximum, minimum, and range of Terri’s math scores.

<table>
<thead>
<tr>
<th>1st of Terri’s Scores</th>
<th>2nd Set of Terri’s Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Maximum</td>
<td>d. Maximum</td>
</tr>
<tr>
<td>b. Minimum</td>
<td>e. Minimum</td>
</tr>
<tr>
<td>c. Range</td>
<td>f. Range</td>
</tr>
</tbody>
</table>

10. As a class, create a list of the heights of all the students.

<table>
<thead>
<tr>
<th>Name</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Determine the mean height of the class.

___________________________________________________________________________________

b. Determine the median.

___________________________________________________________________________________

c. Determine the mode.

___________________________________________________________________________________

d. Determine the maximum.

___________________________________________________________________________________

e. Determine the minimum.

___________________________________________________________________________________

f. Determine the range.

___________________________________________________________________________________

g. What does the mean of this data tell us?

___________________________________________________________________________________
___________________________________________________________________________________
___________________________________________________________________________________
___________________________________________________________________________________
Use the graph below to answer the following questions.

**Principal Languages of the World**

1. Approximately how many people speak these languages?
   a. Mandarin ______________________  b. English ______________________
2. For each person who speaks French, how many speak:
3. How is the data spread out? What does the spread indicate?

   ____________________________________________________________________
   ____________________________________________________________________
4. What other observations can you make about the data?

   ____________________________________________________________________
   ____________________________________________________________________
5. Who might be interested in this type of information?

   ____________________________________________________________________
   ____________________________________________________________________
1. Determine the mean, median, and mode of the life expectancy data for the countries given.
   a. Mean ________________
   b. Median ________________
   c. Mode ________________

2. What does the mean of the life expectancy data tell us?

   __________________________________________
   __________________________________________
   __________________________________________

3. Determine the range of the life expectancy data.

   __________________________________________

<table>
<thead>
<tr>
<th>Country</th>
<th>Life Expectancy at Birth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andorra</td>
<td>87</td>
</tr>
<tr>
<td>Argentina</td>
<td>79</td>
</tr>
<tr>
<td>Australia</td>
<td>83</td>
</tr>
<tr>
<td>Brazil</td>
<td>75</td>
</tr>
<tr>
<td>Burundi</td>
<td>44</td>
</tr>
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<tr>
<td>Japan</td>
<td>84</td>
</tr>
<tr>
<td>Kenya</td>
<td>46</td>
</tr>
<tr>
<td>Mexico</td>
<td>75</td>
</tr>
<tr>
<td>Netherlands</td>
<td>82</td>
</tr>
<tr>
<td>Nigeria</td>
<td>52</td>
</tr>
<tr>
<td>Peru</td>
<td>73</td>
</tr>
<tr>
<td>Poland</td>
<td>78</td>
</tr>
<tr>
<td>Puerto Rico</td>
<td>81</td>
</tr>
<tr>
<td>Russia</td>
<td>73</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>81</td>
</tr>
<tr>
<td>United States</td>
<td>80</td>
</tr>
<tr>
<td>Venezuela</td>
<td>77</td>
</tr>
</tbody>
</table>

Source: www.census.gov
4. Jauntea's math class created a stack plot of the number of siblings of each member of the class. What do the four x's above the 3 represent?

5. Determine the mean, median, and mode of the data.

   a. Mean ________

   b. Median ________

   c. Mode ________

6. What does the mean of the stack plot data tell us? ________________________________
Outcome Sentences

I can now describe the difference between the mean and the median which is, _____________

________________________________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________

We used statistics in our class to ______________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________

In order to find the median, you must first _______________________________________________

________________________________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________

I still have difficulty understanding ______________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________
The following is an example of the Teacher’s Manual to the same lesson from Unit 1.

Transition to Advanced Mathematics Teacher’s Manual does not merely contain answers, but also valuable insights and instructional strategies to use to make the lesson a success.

Below is a brief overview of how each lesson in the Teacher’s Manual is laid out.

• Each lesson opens with a brief description of the lesson. The opening page outlines each lesson’s objectives and the necessary tools and materials needed for the lesson.

• Before each activity there is a “Teacher Reference” page. This gives teachers an in-depth overview of the activity as well as helpful guidelines, tips, and hints. There is also space available for teachers to make notes on their individual instructional strategies that can be repeated when teaching the lesson.

• The activity pages directly mirror the Student Journal. Each teacher manual references the corresponding Student Journal page number.

• Each lesson concludes with either Outcome Sentences or Closing Activity which gives students the opportunity to reflect upon what they learned. They also give students the opportunity to discover any areas they may still need to give extra attention or receive further teacher instruction.
Lesson 5: Descriptive Statistics

In Brief
Students analyze data in different ways to provide different kinds of information. They examine the spread of data (range, maximum, minimum) and central tendency (median, mean, mode).

Objectives
- Determine the measures of central tendency
- Understand measures of dispersion
- Make judgments and arguments based on graphs and statistics

Shaping the Lesson
- The use of physical models will help students to construct intuitive ideas about the mean.
- Point out that although using models may be tedious at first, it is essential to fully grasp the concept.

Instructional Strategies
- Discussion
- Four Corners
- Counting Off

Tools
- Student Journal

Warm Up
Problems of the Day
Setting the Stage

Suppose a CD's retail price at six stores was:

Better Buy $8.00
Lowmart $6.00
Highmart $10.00
Shopper Plus $7.00
Maxi Waxi $9.00
Music Mania $14.00

Make a bar graph using wooden or plastic cubes for the bars. You can also use Post It-Notes™.

Sketch a picture of the graph on the board as a record.

Have students compute the mean. You may need to review with students.

Discuss with the class what they think the mean tells them about the data.

Compare the process of adding up all the numbers to the process of piling all the cubes into one stack.
Have students rearrange the cubes into equal length bars.

The mean is the number you get if all of the values are evened out or leveled off.

For more practice, make other graphs with cubes, and let students find the mean by leveling out the stacks instead of using numerical computation.
**Discovery Activity**

Begin the activity by leading the class through exercises 1 through 9. Ask students to work with a partner.

Remind students that it is easier to determine the median and mode when the numbers are written in numerical order.

For exercise 10, you may want to use a blank transparency and overhead pens to record the data from the class. You may also want to create a measurement station in each corner of the room.
Graphs and tables give a lot of data. We can summarize that data by producing a few numbers that describe the complete set. These numbers are called statistics. Statistics are used to describe the center of the data and the spread of the data.

Three measures of central tendency are mode, median, and mean.

The mode is the value that occurs most frequently in the set of data. If two or more values are the most frequent, there is more than one mode.

1. Consider Terri’s math scores.

   81, 62, 95, 65, 62, 73, 84, 65, 56, 65, 77, 82, 90

   The mode of this set is **65**

2. Order Terri’s math scores from least to greatest.

   56, 62, 62, 65, 65, 65, 73, 77, 81, 82, 84, 90, 95

   The median is the middle value in an ordered set of data. Half of the values are either above or below the median. If there is an even number of values, the median is the average of the two middle values.

3. The median of this set of Terri’s math scores is **73**

The mean is the sum of all the values in the set of data divided by the total number of values.

4. The mean of this set of Terri’s math scores is

   \[
   \frac{56+62+62+65+65+73+77+81+82+84+90+95}{13} = \frac{957}{13} = 73.6
   \]

   Terri took an additional math test and scored a 100. Terri’s math scores now look as follows:

   81, 62, 95, 65, 62, 73, 84, 65, 56, 65, 77, 82, 90, 100

5. The mode of this set is **65**

6. The median of this set is **75**

7. The mean of Terri’s math scores is **75.5**

   \[
   \frac{56+62+62+65+65+73+77+81+82+84+90+95+100}{14} = \frac{1057}{14} = 75.5
   \]
8. What does the mean of Terri's math scores tell us?

The average score of Terri's math tests is 75.5.

Three ways of measuring spread are the maximum, minimum, and range.

The maximum is the greatest number.

The minimum is the least number.

The range is a single number that tells the difference between the maximum and the minimum values.

9. Determine the maximum, minimum, and range of Terri's math scores.

<table>
<thead>
<tr>
<th>1st of Terri's Scores</th>
<th>2nd Set of Terri's Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Maximum  95</td>
<td>d. Maximum  100</td>
</tr>
<tr>
<td>b. Minimum  56</td>
<td>e. Minimum  56</td>
</tr>
<tr>
<td>c. Range  39</td>
<td>f. Range   44</td>
</tr>
</tbody>
</table>

10. As a class, create a list of the heights of all the students. Answers will vary.

<table>
<thead>
<tr>
<th>Name</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

a. Determine the mean height of the class. Answers will vary.

b. Determine the median. Answers will vary.

c. Determine the mode. Answers will vary.

d. Determine the maximum. Answers will vary.

e. Determine the minimum. Answers will vary.

f. Determine the range. Answers will vary.

g. What does the mean of this data tell us? Sample response: The average height of the class is the mean.
Symbolize It & Math at Work

Use these exercises as class work or as homework.

Something to Think About

Data can be analyzed in different ways to support different kinds of informed decisions. Sound judgments can only be made if we know how data is gathered and analyzed.

Extensions

All the test grades are posted on the overhead or on the board in random order. Each student is to copy the grades on a separate sheet of paper. The students are to determine the range, mean, and median. The next day, the papers are given back to the students, and they can circle where their grades are in relationship to the mean and the median.
Use the graph below to answer the following questions.

**Principal Languages of the World**

<table>
<thead>
<tr>
<th>Language</th>
<th>Number of People in Millions</th>
</tr>
</thead>
<tbody>
<tr>
<td>English</td>
<td>300</td>
</tr>
<tr>
<td>Hindi</td>
<td>250</td>
</tr>
<tr>
<td>Arabic</td>
<td>150</td>
</tr>
<tr>
<td>Portuguese</td>
<td>120</td>
</tr>
<tr>
<td>Mandarin</td>
<td>900</td>
</tr>
<tr>
<td>Russian</td>
<td>120</td>
</tr>
<tr>
<td>Spanish</td>
<td>120</td>
</tr>
<tr>
<td>French</td>
<td>100</td>
</tr>
<tr>
<td>Bengali</td>
<td>100</td>
</tr>
</tbody>
</table>

1. Approximately how many people speak these languages?
   a. Mandarin **890 million**
   b. English **440 million**

2. For each person who speaks French, how many speak:
   a. Russian? **About 3**
   b. Arabic? **About 2**

3. How is the data spread out? What does the spread indicate?

   *Sample response: There is a large difference between Mandarin and all the other languages.*

4. What other observations can you make about the data?

   *Sample response: For each person who speaks French, about 8 people speak Mandarin.*

5. Who might be interested in this type of information?

   *Sample response: Governments*
1. Determine the mean, median, and mode of the life expectancy data for the countries given.

   a. Mean 76
   b. Median 80
   c. Mode 75, 80, 81, 83

2. What does the mean of the life expectancy data tell us?

   The average female life expectancy at birth is 76 years.

3. Determine the range of the life expectancy data. 43

<table>
<thead>
<tr>
<th>Country</th>
<th>Life Expectancy at Birth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andorra</td>
<td>87</td>
</tr>
<tr>
<td>Argentina</td>
<td>79</td>
</tr>
<tr>
<td>Australia</td>
<td>83</td>
</tr>
<tr>
<td>Brazil</td>
<td>75</td>
</tr>
<tr>
<td>Burundi</td>
<td>44</td>
</tr>
<tr>
<td>Canada</td>
<td>83</td>
</tr>
<tr>
<td>Chile</td>
<td>80</td>
</tr>
<tr>
<td>Colombia</td>
<td>75</td>
</tr>
<tr>
<td>Cuba</td>
<td>80</td>
</tr>
<tr>
<td>France</td>
<td>83</td>
</tr>
<tr>
<td>Germany</td>
<td>81</td>
</tr>
<tr>
<td>Hungary</td>
<td>77</td>
</tr>
<tr>
<td>Italy</td>
<td>82</td>
</tr>
<tr>
<td>Japan</td>
<td>84</td>
</tr>
<tr>
<td>Kenya</td>
<td>46</td>
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<tr>
<td>Mexico</td>
<td>75</td>
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<td>Netherlands</td>
<td>82</td>
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<td>Puerto Rico</td>
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<td>Russia</td>
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<tr>
<td>United Kingdom</td>
<td>81</td>
</tr>
<tr>
<td>United States</td>
<td>80</td>
</tr>
<tr>
<td>Venezuela</td>
<td>77</td>
</tr>
</tbody>
</table>

Source: www.census.gov
4. Jauntee's math class created a stack plot of the number of siblings of each member of the class. What do the four x's above the 3 represent? *Four people in the class have three siblings.*

<table>
<thead>
<tr>
<th>Number of Siblings of Each Class Member</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
</tr>
<tr>
<td>x</td>
</tr>
<tr>
<td>x x</td>
</tr>
<tr>
<td>x x</td>
</tr>
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<td>x x</td>
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<td>x x x</td>
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<td>x x x</td>
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<td>x x x x</td>
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<tr>
<td>x x x</td>
</tr>
<tr>
<td>x x x</td>
</tr>
<tr>
<td>x x x</td>
</tr>
<tr>
<td>0 1 2 3 4 5 6 7</td>
</tr>
</tbody>
</table>

5. Determine the mean, median, and mode of the data.

   a. Mean
   \[
   \frac{(0 \cdot 3) + (1 \cdot 9) + (2 \cdot 7) + (3 \cdot 4) + (4 \cdot 1) + (5 \cdot 2) + (7 \cdot 2)}{28} = 2.25
   \]

   b. Median 2
   
   \[0, 0, 1, 1, 1, 1, 1, 1, 2, 2 \ | 2, 2, 2, 2, 3, 3, 3, 4, 5, 5, 7, 7\]

   c. Mode 1 *(Most people have one sibling)*

6. What does the mean of the stack plot data tell us?

   *The average number of siblings is 2.25.*
**Outcome Sentences**

I can now describe the difference between the mean and the median which is, ____________

________________________________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________

We used statistics in our class to __________________________________________________________________

________________________________________________________________________________________

________________________________________________________________________________________

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In order to find the median, you must first __________________________________________________________________

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I still have difficulty understanding __________________________________________________________________

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